

Extension of the Poincaré group with half-integer spin generators: hypergravity and beyond

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Abstract

An extension of the Poincaré group with half-integer spin generators is explicitly constructed. We start discussing the case of three spacetime dimensions, and as an application, it is shown that hypergravity can be formulated so as to incorporate this structure as its local gauge symmetry. Since the algebra admits a nontrivial Casimir operator, the theory can be described in terms of gauge fields associated to the extension of the Poincaré group with a Chern-Simons action. The algebra is also shown to admit an infinite-dimensional non-linear extension, that in the case of fermionic spin-3/2 generators, corresponds to a subset of a contraction of two copies of WB_2 . Finally, we show how the Poincaré group can be extended with half-integer spin generators for $d \geq 3$ dimensions.

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I. INTRODUCTION

Nowadays, we have the good fortune of witnessing the era in which the simplest minimal realistic versions of supersymmetric field theories are about to be either tested or falsified by the LHC. The underlying geometric structure of these kind of theories, as well as most of their widely studied extensions, relies on the super-Poincaré group (see, e.g., [1], [2], [3]). According to the Haag-Lopuszański-Sohnius theorem [4], this is a consistent extension of the Poincaré group that includes fermionic generators of spin $1/2$. Indeed, in flat spacetimes of dimension greater than three, the addition of fermionic generators of spin $s \geq 3/2$ would imply that the irreducible representations necessarily contained higher spin fields, which are known to suffer from inconsistencies (see, e.g., [5], [6], [7], [8], [9], [10], [11]). However, in three spacetime dimensions, higher spin fields do not possess local propagating degrees of freedom, and as a consequence, it is possible to describe them consistently [12], [13], [14], [15], [16], [17], [18] even on locally flat spacetimes [19], [20], [21], [22]. Hence, in the latter context, since no-go theorems about massless higher spin fields can be circumvented, it is natural to look for an extension of the Poincaré group with fermionic half-integer spin generators. Results along these lines have already been explored in [23]. In what follows, we begin with the construction of the searched for extension of the Poincaré group in the case of spin $3/2$ generators, that for short, hereafter we dub it the “hyper-Poincaré” group. It is shown that the algebra admits a nontrivial Casimir operator and, as an application, we explain how the hypergravity theory of Aragone and Deser [24] can be formulated so as to incorporate the hyper-Poincaré group as its local gauge symmetry. Concretely, we show how hypergravity can be described in terms of hyper-Poincaré-valued gauge fields with a Chern-Simons action. The results are then extended to the case of fermionic generators of spin $n + \frac{1}{2}$, as well as to the minimal coupling of General Relativity with gauge fields of spin $n + \frac{3}{2}$, so that super-Poincaré group and supergravity are recovered for $n = 0$. The hyper-Poincaré algebra is also shown to admit an infinite-dimensional nonlinear extension that contains the BMS_3 algebra, which in the case of spin- $3/2$ generators, reduces to a subset of a suitable contraction of two copies of WB_2 . We conclude explaining how the hyper-Poincaré group is extended to the case of $d \geq 3$ dimensions.

II. FERMIONIC SPIN-3/2 GENERATORS

In three spacetime dimensions, the nonvanishing commutators of the Poincaré algebra can be written as

$$[J_a, J_b] = \varepsilon_{abc} J^c \quad , \quad [J_a, P_b] = \varepsilon_{abc} P^c . \quad (1)$$

The additional fermionic generators are assumed to transform in an irreducible spin-3/2 representation of the Lorentz group, so that they are described by “ Γ -traceless” vector-spinors that fulfill $Q^a \Gamma_a = 0$, where Γ_a stand for the Dirac matrices. Their corresponding commutation rules with the Lorentz generators are then given by

$$[J_a, Q_{ab}] = \frac{1}{2} (\Gamma_a)^\beta{}_\alpha Q_{\beta b} + \varepsilon_{abc} Q_\alpha^c . \quad (2)$$

Therefore, requiring consistency of the closure as well as the Jacobi identity, implies that the only remaining nonvanishing (anti-) commutators of the algebra read

$$\{Q_\alpha^a, Q_\beta^b\} = -\frac{2}{3} (C\Gamma^c)_{\alpha\beta} P_c \eta^{ab} + \frac{5}{6} \varepsilon^{abc} C_{\alpha\beta} P_c + \frac{1}{6} (C\Gamma^a)_{\alpha\beta} P^b , \quad (3)$$

where C is the charge conjugation matrix¹. It is then simple to verify that apart from $I_1 = P^a P_a$, the algebra admits another Casimir operator given by

$$I_2 = 2J^a P_a + Q_\alpha^a C^{\alpha\beta} Q_{\beta a} , \quad (4)$$

which implies the existence of an invariant (anti-) symmetric bilinear form, whose only nonvanishing components are of the form

$$\begin{aligned} \langle J_a, P_b \rangle &= \eta_{ab} , \\ \langle Q_\alpha^a, Q_\beta^b \rangle &= \frac{2}{3} C_{\alpha\beta} \eta^{ab} - \frac{1}{3} \varepsilon^{abc} (C\Gamma_c)_{\alpha\beta} . \end{aligned} \quad (5)$$

It is worth highlighting that the inclusion of the higher spin generators Q_α^a does not jeopardize the causal structure, since there is no need to enlarge the Lorentz group.

¹ In our conventions, the Minkowski metric η_{ab} is assumed to follow the “mostly plus” convention, and the Levi-Civita symbol fulfills $\varepsilon_{012} = 1$. Round brackets stand for symmetrization of the enclosed indices, without the normalization factor, e. g., $X^{(a} Y^b Z^{c)} = X^a Y^b Z^c + X^c Y^b Z^a$. It is also useful to keep in mind the Fierz expansion of the product of three Dirac matrices, given by $\Gamma^a \Gamma^b \Gamma^c = \varepsilon^{abc} + \eta^{ab} \Gamma^c + \eta^{bc} \Gamma^a - \eta^{ac} \Gamma^b$. Afterwards, the presence of the imaginary unit “ i ” in the product of real Grassmann variables is because we assume that $(\theta_1 \theta_2)^* = -\theta_1 \theta_2$.

A. Hypergravity

In order to describe a massless spin- $\frac{5}{2}$ field minimally coupled to General Relativity, let us consider a connection 1-form that takes values in the hyper-Poincaré algebra described above, which reads

$$A = e^a P_a + \omega^a J_a + \psi_a^\alpha Q_\alpha^a, \quad (6)$$

where e^a , ω^a and ψ_a^α stand for the dreibein, the dualized spin connection ($\omega^a = \frac{1}{2}\varepsilon^{abc}\omega_{bc}$), and the Γ -traceless spin- $\frac{5}{2}$ field ($\Gamma^a\psi_a = 0$), respectively. The components of the field strength $F = dA + A^2$ are then given by

$$F = R^a J_a + \tilde{T}^a P_a + D\psi_a^\alpha Q_\alpha^a, \quad (7)$$

where the covariant derivative of the spin- $\frac{5}{2}$ field can be written as $D\psi^a = d\psi^a + \frac{3}{2}\omega^b\Gamma_b\psi^a - \omega_b\Gamma^a\psi^b$, and $R^a = d\omega^a + \frac{1}{2}\varepsilon^{abc}\omega_b\omega_c$ is the dualized curvature 2-form. The hypercovariant torsion 2-form then reads

$$\tilde{T}^a := T^a - \frac{3}{4}i\bar{\psi}_b\Gamma^a\psi^b, \quad (8)$$

with $T^a = de^a + \varepsilon^{abc}\omega_b e_c$, and $\bar{\psi}_{a\alpha} = \psi_a^\beta C_{\beta\alpha}$ is the Majorana conjugate.

Note that under an infinitesimal gauge transformation $\delta A = d\lambda + [A, \lambda]$, spanned by a hyper-Poincaré-valued zero-form given by $\lambda = \lambda^a P_a + \sigma^a J_a + \epsilon_a^\alpha Q_\alpha^a$, the components of the gauge field transform according to

$$\begin{aligned} \delta e^a &= D\lambda^a - \varepsilon^{abc}\sigma_b e_c + \frac{3}{2}i\bar{\epsilon}_b\Gamma^a\psi^b, \\ \delta\omega^a &= D\sigma^a, \\ \delta\psi^a &= -\frac{3}{2}\sigma^b\Gamma_b\psi^a + \sigma_b\Gamma^a\psi^b + D\epsilon^a. \end{aligned} \quad (9)$$

The invariant bilinear form (5) then allows to construct a Chern-Simons action for the gauge field (6), given by

$$I = \frac{k}{4\pi} \int \left\langle AdA + \frac{2}{3}A^3 \right\rangle, \quad (10)$$

which up to a boundary term, reduces to

$$I = \frac{k}{4\pi} \int 2R^a e_a + i\bar{\psi}_a D\psi^a. \quad (11)$$

It is worth pointing out that, despite the action (11) is formally the same as the one considered by Aragone and Deser in [24], it does possess a different local structure. Indeed, note

that under local hypersymmetry transformations spanned by $\lambda = \epsilon_a^\alpha Q_\alpha^a$, the nonvanishing transformation rule for the spin connection considered in [24], agrees with ours only on-shell. Actually, by construction, as in the case of supergravity [25], here the algebra of the local gauge symmetries (9) closes off-shell according to the hyper-Poincaré group, without the need of auxiliary fields.

In the case of negative cosmological constant, it can be seen that hypergravity requires the presence of additional spin-4 fields [26], [27], [28].

III. FERMIONIC GENERATORS OF SPIN $s = n + \frac{1}{2}$

In this case, the fermionic generators correspond to tensor-spinors $Q_\alpha^{a_1 \dots a_n}$, transforming in an irreducible representation of the Lorentz group, so that they are completely symmetric in the vector indices, as well as Γ -traceless, i. e., $Q^{a_1 \dots a_n} \Gamma_{a_1} = 0$. These conditions imply that their anticommutation rules acquire a somehow cumbersome expression, and it is then more convenient to write the hyper-Poincaré algebra in the Maurer-Cartan formalism. The Maurer-Cartan 1-form is given by

$$\Omega = \rho^a P_a + \tau^a J_a + \chi_{a_1 \dots a_n}^\alpha Q_\alpha^{a_1 \dots a_n}, \quad (12)$$

where $\chi_{a_1 \dots a_n}$ is Γ -traceless and completely symmetric in the vector indices, which can be seen as a flat connection that fulfills

$$\begin{aligned} d\tau^a &= -\frac{1}{2}\epsilon^{abc}\tau_b\tau_c, \\ d\rho^a &= -\epsilon^{abc}\tau_b\rho_c + \frac{1}{2}\left(n + \frac{1}{2}\right)i\bar{\chi}_{a_1 \dots a_n}\Gamma^a\chi^{a_1 \dots a_n}, \\ d\chi^{a_1 \dots a_n} &= -\left(n + \frac{1}{2}\right)\tau_b\Gamma^b\chi^{a_1 \dots a_n} + \tau_b\Gamma^{(a_1}\chi^{a_2 \dots a_n)b}. \end{aligned} \quad (13)$$

Note that the Jacobi identity now translates into the consistency of the nilpotence of the exterior derivative ($d^2 = 0$), which for the algebra (13) is clearly satisfied.

The nontrivial Casimir operator now reads

$$I_2 = 2J^a P_a + Q_{\alpha a_1 \dots a_n} C^{\alpha\beta} Q_\beta^{a_1 \dots a_n}. \quad (14)$$

It is also worth pointing out that the super-Poincaré algebra corresponds to the case of $n = 0$, while the hyper-Poincaré algebra described above is recovered for $n = 1$.

A. Hypergravity in the generic case

The minimal coupling of General Relativity with a massless fermionic field of spin $s = n + \frac{3}{2}$, described by a completely symmetric Γ -traceless 1-form $\psi_{a_1 \dots a_n}$, can then be formulated in terms of a gauge field for the hyper-Poincaré algebra, which now reads

$$A = e^a P_a + \omega^a J_a + \psi_{a_1 \dots a_n}^\alpha Q_\alpha^{a_1 \dots a_n} . \quad (15)$$

The components of the curvature 2-form are then given by

$$F = R^a J_a + \tilde{T}^a P_a + D\psi_{a_1 \dots a_n}^\alpha Q_\alpha^{a_1 \dots a_n} , \quad (16)$$

where the covariant derivative of the spin- $(n + \frac{3}{2})$ field can be written as

$$D\psi^{a_1 \dots a_n} = d\psi^{a_1 \dots a_n} + \left(n + \frac{1}{2}\right) \omega_b \Gamma^b \psi^{a_1 \dots a_n} - \omega_b \Gamma^{(a_1} \psi^{a_2 \dots a_n) b} , \quad (17)$$

and

$$\tilde{T}^a = T^a - \frac{1}{2} \left(n + \frac{1}{2}\right) i \bar{\psi}_{a_1 \dots a_n} \Gamma^a \psi^{a_1 \dots a_n} . \quad (18)$$

The transformation rules of the fields under local hypersymmetry can then be obtained from a gauge transformation of the connection (15) with a fermionic parameter given by $\lambda = \epsilon_{a_1 \dots a_n}^\alpha Q_\alpha^{a_1 \dots a_n}$, so that they read

$$\begin{aligned} \delta e^a &= \left(n + \frac{1}{2}\right) i \bar{\epsilon}_{a_1 \dots a_n} \Gamma^a \psi^{a_1 \dots a_n} , \\ \delta \omega^a &= 0 , \\ \delta \psi^{a_1 \dots a_n} &= D \epsilon^{a_1 \dots a_n} . \end{aligned} \quad (19)$$

The Casimir operator (14) then implies the existence of an (anti-) symmetric tensor of rank 2, which once contracted with the wedge product of two curvatures, gives

$$\begin{aligned} \langle F^2 \rangle &= 2R^a \tilde{T}_a + i D \bar{\psi}_{a_1 \dots a_n} D \psi^{a_1 \dots a_n} \\ &= d \left(2R^a e_a + i \bar{\psi}_{a_1 \dots a_n} D \psi^{a_1 \dots a_n} \right) , \end{aligned} \quad (20)$$

being an exact form that is manifestly invariant under the hypersymmetry transformations (19). Therefore, as in the case of (super)gravity [29], [30], the action can also be written as a Chern-Simons one (10), which up to a boundary term reduces to

$$I = \frac{k}{4\pi} \int 2R^a e_a + i \bar{\psi}_{a_1 \dots a_n} D \psi^{a_1 \dots a_n} , \quad (21)$$

so that the field equations now read $F = 0$, with F given by (16).

Note that the standard supergravity action in [31], [32], [33] is recovered for $n = 0$; and as it occurs in the spin- $\frac{5}{2}$ case, the generic theory agrees with the one of Aragone and Deser only on-shell.

We would like to stress that a deeper understanding of the theory cannot be attained unless it is endowed with a consistent set of boundary conditions. In this sense, one of the advantages of formulating hypergravity as a Chern-Simons theory is that the analysis of its asymptotic structure can be directly performed in a canonical form, as in the case of negative cosmological constant [28]. Indeed, in analogy with the case of three-dimensional flat supergravity [34], the mode expansion of the asymptotic symmetry algebra of hypergravity with a spin- $\frac{5}{2}$ fermionic field is defined through the following Poisson brackets [35]

$$\begin{aligned}
i \{ \mathcal{J}_m, \mathcal{J}_n \} &= (m - n) \mathcal{J}_{m+n}, \\
i \{ \mathcal{J}_m, \mathcal{P}_n \} &= (m - n) \mathcal{P}_{m+n} + km (m^2 - 1) \delta_{m+n,0}, \\
i \{ \mathcal{P}_m, \mathcal{P}_n \} &= 0 \quad , \quad i \{ \mathcal{P}_m, \psi_n \} = 0, \\
i \{ \mathcal{J}_m, \psi_n \} &= \left(\frac{3m}{2} - n \right) \psi_{m+n}, \\
i \{ \psi_m, \psi_n \} &= \frac{1}{4} (6m^2 - 8mn + 6n^2 - 9) \mathcal{P}_{m+n} + \frac{9}{4k} \sum_q \mathcal{P}_{m+n-q} \mathcal{P}_q \\
&\quad + k \left(m^2 - \frac{9}{4} \right) \left(n^2 - \frac{1}{4} \right) \delta_{m+n,0},
\end{aligned} \tag{22}$$

which describe a nonlinear hypersymmetric extension of the BMS_3 algebra [36], [37], [38]. It can also be shown that this algebra corresponds to a subset of a suitable contraction of two copies of the WB_2 algebra [39], [40], [28].

When fermions fulfill antiperiodic boundary conditions, the modes of the fermionic global charges ψ_m are labelled by half-integers, so that the wedge algebra of (22) reduces to the one of the hyper-Poincaré group. In fact, dropping the nonlinear terms, and restricting the modes according to $|n| < \Delta$, where Δ stands for the conformal weight of the generators, the hyper-Poincaré algebra is manifestly recovered provided the modes in (22) are identified

with the generators $J_a, P_a, Q_{\alpha a}$, according to

$$\begin{aligned}
\mathcal{J}_{-1} &= -2J_0 \quad , \quad \mathcal{J}_1 = J_1 \quad , \quad \mathcal{J}_0 = J_2 \quad , \\
\mathcal{P}_{-1} &= -2P_0 \quad , \quad \mathcal{P}_1 = P_1 \quad , \quad \mathcal{P}_0 = P_2 \quad , \\
\psi_{-\frac{3}{2}} &= 2^{\frac{5}{4}}\sqrt{3}Q_{+0} \quad , \quad \psi_{-\frac{1}{2}} = 2^{\frac{3}{4}}\sqrt{3}Q_{-0} \quad , \\
\psi_{\frac{1}{2}} &= -2^{\frac{1}{4}}\sqrt{3}Q_{+1} \quad , \quad \psi_{\frac{3}{2}} = -2^{-\frac{1}{4}}\sqrt{3}Q_{-1} \quad .
\end{aligned} \tag{23}$$

It is also worth noting that (22) can then be regarded as a hypersymmetric extension of the Galilean conformal algebra in two dimensions [41], [42], which is isomorphic to BMS_3 and turns out to be relevant in the context of non-relativistic holography.

Another advantage of formulating hypergravity in terms of a Chern-Simons action is that, as in case of supergravity [43], [34], the theory can be readily extended to include parity odd terms in the Lagrangian. This can be explicitly performed by a simple modification of the invariant bilinear form, so that it acquires an additional component given by $\langle J_a, J_b \rangle = \mu \eta_{ab}$, followed by a shift in the spin connection of the form $\omega^a \rightarrow \omega^a + \gamma e^a$, so that the constants μ, γ parametrize the new allowed couplings in the action. As a consequence, when hypergravity is extended in this way, the hyper- BMS_3 algebra (22) acquires an additional nontrivial central extension along its Virasoro subgroup.

IV. ENDING REMARKS

The hyper-Poincaré group admits a consistent generalization to the case of $d \geq 3$ space-time dimensions. In the case of fermionic Γ -traceless spin- $\frac{3}{2}$ generators, the nonvanishing (anti-) commutators of the algebra are given by

$$\begin{aligned}
[J_{ab}, J_{cd}] &= J_{ad}\eta_{bc} - J_{bd}\eta_{ac} + J_{ca}\eta_{ad} \quad , \\
[J_{ab}, P_c] &= P_a\eta_{bc} - P_b\eta_{ac} \quad , \\
[J_{ab}, Q_c^\alpha] &= -\frac{1}{2}(\Gamma_{ab})^\alpha{}_\beta Q_c^\beta + Q_a^\alpha\eta_{bc} - Q_b^\alpha\eta_{ac} \quad , \\
[J_{ab}, \bar{Q}_{\alpha c}] &= \frac{1}{2}(\Gamma_{ab})^\beta{}_\alpha \bar{Q}_{\beta c} + \bar{Q}_{\alpha a}\eta_{bc} - \bar{Q}_{\alpha b}\eta_{ac} \quad , \\
\{Q^{\alpha a}, \bar{Q}_\beta^b\} &= \frac{3(d-2)}{d^2}i \left[(d+1)(\Gamma^c)^\alpha{}_\beta P_c \eta^{ab} - \frac{d+2}{d-2}(\Gamma^{abc})^\alpha{}_\beta P_c - (\Gamma^{(a|})^\alpha{}_\beta P^{b|)} \right] \quad ,
\end{aligned} \tag{24}$$

where $\bar{Q}_a = Q_a^\dagger \Gamma^0$ stands for the Dirac conjugate.

In the generic case, the spin- $(n + \frac{1}{2})$ generators correspond to completely symmetric Γ -traceless tensor-spinors that fulfill $\Gamma^{a_1} Q_{a_1 \dots a_n} = 0$. In order to avoid the intricacies related to the latter condition, as well as with the suitable (anti-) symmetrization of the (anti-) commutation rules of the generators, it is better to express the algebra in terms of its Maurer-Cartan form. It is now given by

$$\Omega = \rho^a P_a + \frac{1}{2} \tau^{ab} J_{ab} + \bar{\chi}_{\alpha}^{a_1 \dots a_n} Q_{a_1 \dots a_n}^{\alpha} - \bar{Q}_{\alpha}^{a_1 \dots a_n} \chi_{a_1 \dots a_n}^{\alpha}, \quad (25)$$

where $\chi_{a_1 \dots a_n}$ is Γ -traceless and completely symmetric in the vector indices, so that its components fulfill ²

$$\begin{aligned} d\tau^{ab} &= -\tau^a{}_c \tau^{cb}, \\ d\rho^a &= -\tau^a{}_b \rho^b + \frac{1}{2} \left(n + \frac{1}{2} \right) i \bar{\chi}_{a_1 \dots a_n} \Gamma^a \chi^{a_1 \dots a_n}, \\ d\chi^{a_1 \dots a_n} &= -\frac{1}{4} \tau^{ab} \Gamma_{ab} \chi^{a_1 \dots a_n} - \tau^{(a_1}{}_b \chi^{a_2 \dots a_n)b}, \\ d\bar{\chi}^{a_1 \dots a_n} &= -\frac{1}{4} \bar{\chi}^{a_1 \dots a_n} \tau^{ab} \Gamma_{ab} - \tau^{(a_1}{}_b \bar{\chi}^{a_2 \dots a_n)b}. \end{aligned} \quad (26)$$

This algebra can be easily written in terms of Majorana spinors when they exist, and it reduces to super-Poincaré for $n = 0$.

Note that there was no need to enlarge the Lorentz group in order to accommodate the higher spin generators, so that the additional symmetries do not seem to interfere with the causal structure. Indeed, as in the case of supersymmetry, the quotient of the hyper-Poincaré group over the Lorentz subgroup now defines a hyperspace which is an extension of Minkowski spacetime with additional Γ -traceless tensor-spinor coordinates. However, as anticipated by Haag, Łopuszański and Sohnius, the irreducible representations, which could be obtained from suitable hyperfields, necessarily contain higher spin fields. Nevertheless, it would be worth to explore whether the hyper-Poincaré algebra may manifest itself through theories or models whose fundamental fields do not transform as linear multiplets, as it would be the case of nonlinear realizations, hyper-Poincaré-valued gauge fields, or extended objects.

² In the case of $d = 2$ spacetime dimensions the algebra is consistent. However, the subset spanned by translations and the fermionic generators is an abelian ideal.

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- [1] P. Fayet and S. Ferrara, “Supersymmetry,” Phys. Rept. **32**, 249 (1977).
 - [2] M. F. Sohnius, “Introducing Supersymmetry,” Phys. Rept. **128**, 39 (1985).
 - [3] P. Van Nieuwenhuizen, “Supergravity,” Phys. Rept. **68**, 189 (1981).
 - [4] R. Haag, J. T. Lopuszanski and M. Sohnius, “All Possible Generators of Supersymmetries of the s Matrix,” Nucl. Phys. B **88**, 257 (1975).
 - [5] S. Weinberg, “Photons and Gravitons in s Matrix Theory: Derivation of Charge Conservation and Equality of Gravitational and Inertial Mass,” Phys. Rev. **135**, B1049 (1964).
 - [6] C. Aragone and S. Deser, “Consistency Problems of Hypergravity,” Phys. Lett. B **86**, 161 (1979).
 - [7] F. A. Berends, J. W. van Holten, P. van Nieuwenhuizen and B. de Wit, “On Spin 5/2 Gauge Fields,” Phys. Lett. B **83**, 188 (1979) [Phys. Lett. **84B**, 529 (1979)].
 - [8] C. Aragone and S. Deser, “Higher Spin Vierbein Gauge Fermions and Hypergravities,” Nucl. Phys. B **170**, 329 (1980).
 - [9] S. Weinberg and E. Witten, “Limits on Massless Particles,” Phys. Lett. B **96**, 59 (1980).
 - [10] M. Porrati, “Universal Limits on Massless High-Spin Particles,” Phys. Rev. D **78**, 065016 (2008) [arXiv:0804.4672 [hep-th]].
 - [11] X. Bekaert, N. Boulanger and P. Sundell, “How higher-spin gravity surpasses the spin two barrier: no-go theorems versus yes-go examples,” Rev. Mod. Phys. **84**, 987 (2012) [arXiv:1007.0435 [hep-th]].
 - [12] M. P. Blencowe, “A Consistent Interacting Massless Higher Spin Field Theory in $D = (2+1)$,” Class. Quant. Grav. **6**, 443 (1989).

- [13] E.Bergshoeff, M.P.Blencowe and K.S.Stelle, “Area Preserving Diffeomorphisms and Higher Spin Algebra,” Commun. Math. Phys. **128**,213 (1990).
- [14] M. A. Vasiliev, “Higher spin gauge theories in four-dimensions, three-dimensions, and two-dimensions,” Int. J. Mod. Phys. D **5**, 763 (1996) [hep-th/9611024].
- [15] M. Henneaux and S. J. Rey, “Nonlinear $W_{infinity}$ as Asymptotic Symmetry of Three-Dimensional Higher Spin Anti-de Sitter Gravity,” JHEP **1012**, 007 (2010) [arXiv:1008.4579 [hep-th]].
- [16] A. Campoleoni, S. Fredenhagen, S. Pfenninger and S. Theisen, “Asymptotic symmetries of three-dimensional gravity coupled to higher-spin fields,” JHEP **1011**, 007 (2010) [arXiv:1008.4744 [hep-th]].
- [17] M. Henneaux, A. Perez, D. Tempo and R. Troncoso, “Chemical potentials in three-dimensional higher spin anti-de Sitter gravity,” JHEP **1312**, 048 (2013) [arXiv:1309.4362 [hep-th]].
- [18] C. Bunster, M. Henneaux, A. Perez, D. Tempo and R. Troncoso, “Generalized Black Holes in Three-dimensional Spacetime,” JHEP **1405**, 031 (2014) [arXiv:1404.3305 [hep-th]].
- [19] H. Afshar, A. Bagchi, R. Fareghbal, D. Grumiller and J. Rosseel, “Spin-3 Gravity in Three-Dimensional Flat Space,” Phys. Rev. Lett. **111**, 121603 (2013) [arXiv:1307.4768 [hep-th]].
- [20] H. A. Gonzalez, J. Matulich, M. Pino and R. Troncoso, “Asymptotically flat spacetimes in three-dimensional higher spin gravity,” JHEP **1309** (2013) 016 [arXiv:1307.5651 [hep-th]].
- [21] M. Gary, D. Grumiller, M. Riegler and J. Rosseel, “Flat space (higher spin) gravity with chemical potentials,” JHEP **1501**, 152 (2015) [arXiv:1411.3728 [hep-th]].
- [22] J. Matulich, A. Perez, D. Tempo and R. Troncoso, “Higher spin extension of cosmological spacetimes in 3D: asymptotically flat behaviour with chemical potentials and thermodynamics,” JHEP **1505**, 025 (2015) [arXiv:1412.1464 [hep-th]].
- [23] J. Hietarinta, “Supersymmetry Generators of Arbitrary Spin,” Phys. Rev. D **13**, 838 (1976).
- [24] C. Aragone and S. Deser, “Hypersymmetry in $D = 3$ of Coupled Gravity Massless Spin 5/2 System,” Class. Quant. Grav. **1**, L9 (1984).
- [25] M. Banados, R. Troncoso and J. Zanelli, “Higher dimensional Chern-Simons supergravity,” Phys. Rev. D **54**, 2605 (1996) [gr-qc/9601003].
- [26] B. Chen, J. Long and Y. N. Wang, “Conical Defects, Black Holes and Higher Spin (Super-)Symmetry,” JHEP **1306**, 025 (2013) [arXiv:1303.0109 [hep-th]].
- [27] Y. M. Zinoviev, “Hypergravity in AdS_3 ,” Phys. Lett. B **739**, 106 (2014) [hep-th/1408.2912].

- [28] M. Henneaux, A. Perez, D. Tempo and R. Troncoso, “Hypersymmetry bounds and three-dimensional higher-spin black holes,” *JHEP* **1508**, 021 (2015) [arXiv:1506.01847 [hep-th]].
- [29] A. Achucarro and P. K. Townsend, “A Chern-Simons Action for Three-Dimensional anti-De Sitter Supergravity Theories,” *Phys. Lett. B* **180**, 89 (1986).
- [30] E. Witten, “(2+1)-Dimensional Gravity as an Exactly Soluble System,” *Nucl. Phys. B* **311**, 46 (1988).
- [31] S. Deser and J. H. Kay, “Topologically Massive Supergravity,” *Phys. Lett. B* **120**, 97 (1983).
- [32] S. Deser, “Cosmological Topological Supergravity,” In *Christensen, S.m. (Ed.): Quantum Theory Of Gravity*, 374-381
- [33] N. Marcus and J. H. Schwarz, “Three-Dimensional Supergravity Theories,” *Nucl. Phys. B* **228**, 145 (1983).
- [34] G. Barnich, L. Donnay, J. Matulich and R. Troncoso, “Asymptotic symmetries and dynamics of three-dimensional flat supergravity,” *JHEP* **1408**, 071 (2014) [arXiv:1407.4275 [hep-th]].
- [35] O. Fuentealba, J. Matulich and R. Troncoso, “Asymptotically flat structure of hypergravity in three spacetime dimensions,” *JHEP* **1510**, 009 (2015) [arXiv:1508.04663 [hep-th]].
- [36] A. Ashtekar, J. Bicak and B. G. Schmidt, “Asymptotic structure of symmetry reduced general relativity,” *Phys. Rev. D* **55**, 669 (1997) [gr-qc/9608042].
- [37] G. Barnich and G. Compere, “Classical central extension for asymptotic symmetries at null infinity in three spacetime dimensions,” *Class. Quant. Grav.* **24**, F15 (2007) [gr-qc/0610130].
- [38] G. Barnich and C. Troessaert, “Aspects of the BMS/CFT correspondence,” *JHEP* **1005**, 062 (2010) [arXiv:1001.1541 [hep-th]].
- [39] J. M. Figueroa-O’Farrill, S. Schrans and K. Thielemans, “On the Casimir algebra of B(2),” *Phys. Lett. B* **263**, 378 (1991).
- [40] S. Bellucci, S. Krivonos and A. S. Sorin, “Linearizing W(2,4) and WB(2) algebras,” *Phys. Lett. B* **347**, 260 (1995) [hep-th/9411168].
- [41] A. Bagchi, “Correspondence between Asymptotically Flat Spacetimes and Nonrelativistic Conformal Field Theories,” *Phys. Rev. Lett.* **105**, 171601 (2010).
- [42] A. Bagchi and R. Gopakumar, “Galilean Conformal Algebras and AdS/CFT,” *JHEP* **0907**, 037 (2009) [arXiv:0902.1385 [hep-th]].
- [43] A. Giacomini, R. Troncoso and S. Willison, “Three-dimensional supergravity reloaded,” *Class. Quant. Grav.* **24**, 2845 (2007) [hep-th/0610077].